Exercise 34

For what values of x does the graph of f have a horizontal tangent?

$$f(x) = e^x \cos x$$

Solution

The graph of f has a horizontal tangent wherever the first derivative is zero. Calculate the first derivative.

$$\begin{aligned} f'(x) &= \frac{d}{dx}[f(x)] \\ &= \frac{d}{dx}(e^x \cos x) \\ &= \left[\frac{d}{dx}(e^x)\right]\cos x + e^x\left[\frac{d}{dx}(\cos x)\right] \\ &= (e^x)\cos x + e^x(-\sin x) \\ &= e^x(\cos x - \sin x) \end{aligned}$$

Set it equal to zero.

$$e^x(\cos x - \sin x) = 0$$

Solve for x.

$$\cos x - \sin x = 0$$

$$\tan x = 1$$

$$x = \left\{\frac{\pi}{4} + n\pi\right\}$$

Here n is any integer.